

**Supplemental Material for BABAR-PUB-20/003***Search for a Dark Leptophilic Scalar in  $e^+e^-$  Collisions*

Additional details and figures for the dark leptophilic scalar search are presented in this Supplemental Material.

TABLE I: List of variables used as input to the dimuon boosted decision trees.

Ratio of second to zeroth Fox-Wolfram moment of all tracks and neutrals.
Invariant mass of the four track system, assuming the pion (muon) mass for the tracks originating from the tau ( $\phi_L$ ) decays.
Invariant mass and transverse momentum of all tracks and neutrals.
Invariant mass squared of the system recoiling against all tracks and neutrals.
Transverse momentum of the system recoiling against all tracks and neutrals.
Number of neutral candidates with an energy greater than 50 MeV.
Invariant masses of the three track systems formed by the $\phi_L$ and the remaining positively or negatively charged tracks.
Momentum of each track from $\phi_L$ decays.
Angle between the two tracks produced by the tau decay.
Variable indicating if a track has been identified as a muon or an electron by PID algorithm for each track.

TABLE II: List of variables used as input to the dielectron boosted decision trees.

Transverse momentum of the system recoiling against all tracks and neutrals.
Energy of the system recoiling against all tracks and neutrals.
Number of tracks identified as electron candidates by a PID algorithm applied to each track.
Angle between $\phi_L$ candidate momentum and closest track produced in tau decay.
Angle between $\phi_L$ candidate momentum and farthest track produced in tau decay.
Angle of $\phi_L$ candidate relative to the beam in the center-of-mass frame.
Angle between the two tracks produced by the tau decay.
Angle between $\phi_L$ candidate and nearest neutral candidate with $E > 50$ MeV.
Energy of nearest neutral candidate (with $E > 50$ MeV) to $\phi_L$ candidate.
Total energy in neutral candidates, each of which has an energy greater than 50 MeV.
Distance between beamspot and $\phi_L$ candidate vertex.
Uncertainty in the distance between beamspot and $\phi_L$ candidate decay vertex.
$\phi_L$ candidate vertex significance, defined by the beamspot-vertex distance divided by its uncertainty.
Angle between the $\phi_L$ candidate momentum, and line from beamspot to $\phi_L$ decay vertex.
Distance of closest approach to beamspot of $e^-$ in $\phi_L$ candidate.
Distance of closest approach to beamspot of $e^+$ in $\phi_L$ candidate.
Transverse distance between $\phi_L$ decay vertex and best-fit common origin of $\tau$ candidates and $\phi_L$ candidate.
$\chi^2$ of the kinematic fit to the $\phi_L$ and $\tau$ candidates constraining their origin to the same production point.
$\chi^2$ of the kinematic fit of the $\phi_L$ candidate with the constraint that the $e^+e^-$ pair is produced from a photon conversion in detector material.
Dielectron mass for $\phi_L$ candidate when re-fit with the photon conversion constraint.

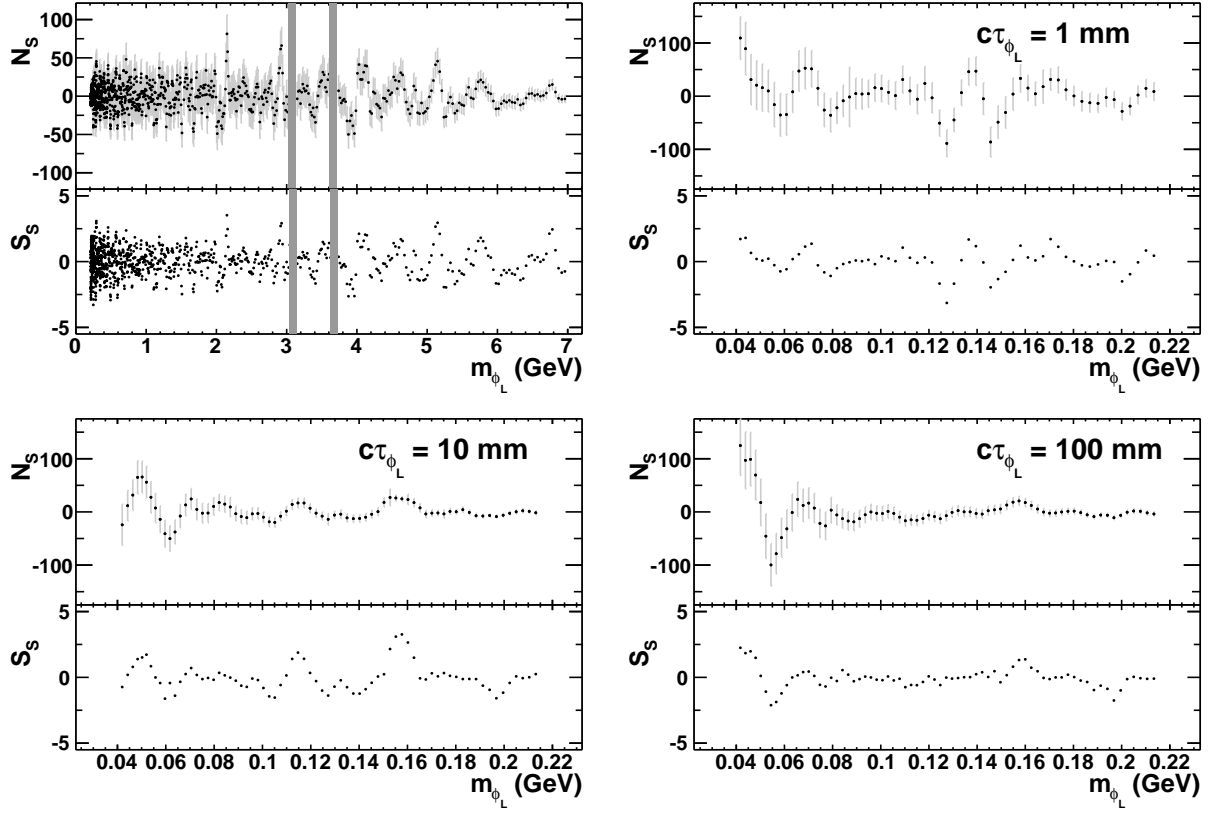


FIG. 1: The distribution of signal events ( $N_s$ ) and local signal significance ( $S_s$ ) from the fits as a function of the  $\phi_L$  mass for (top left) prompt decays; (top right)  $c\tau_{\phi_L} = 1$  mm; (bottom left)  $c\tau_{\phi_L} = 10$  mm; (bottom right)  $c\tau_{\phi_L} = 100$  mm. The prompt decays include contributions from both the dielectron and dimuon final states.

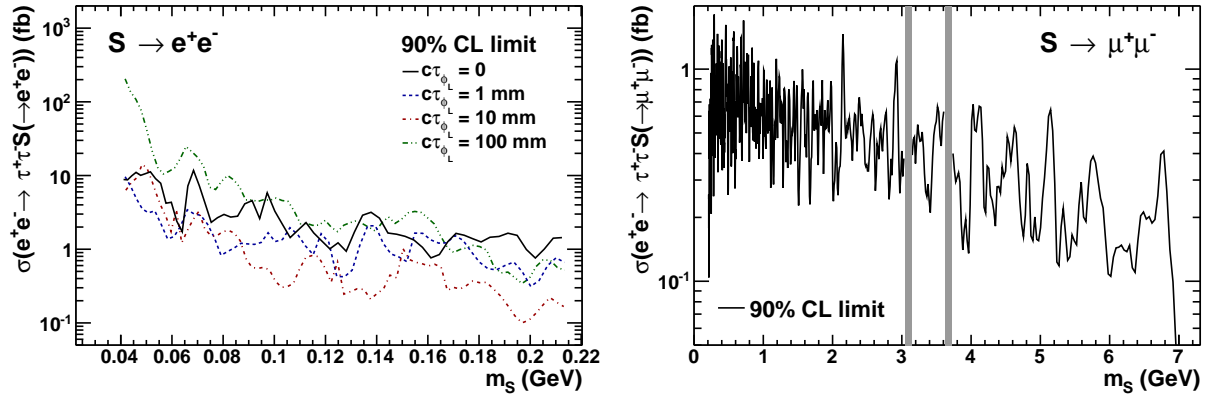


FIG. 2: The 90% CL limits on (left) the  $\sigma(e^+e^- \rightarrow \tau^+\tau^- S(S \rightarrow e^+e^-))$  and (right) the  $\sigma(e^+e^- \rightarrow \tau^+\tau^- S(S \rightarrow \mu^+\mu^-))$  cross sections for the production of a generic scalar  $S$  at the  $\Upsilon(4S)$  resonance. The gray bands indicate the regions excluded from the search around the nominal  $J/\psi$  and  $\psi(2S)$  masses.

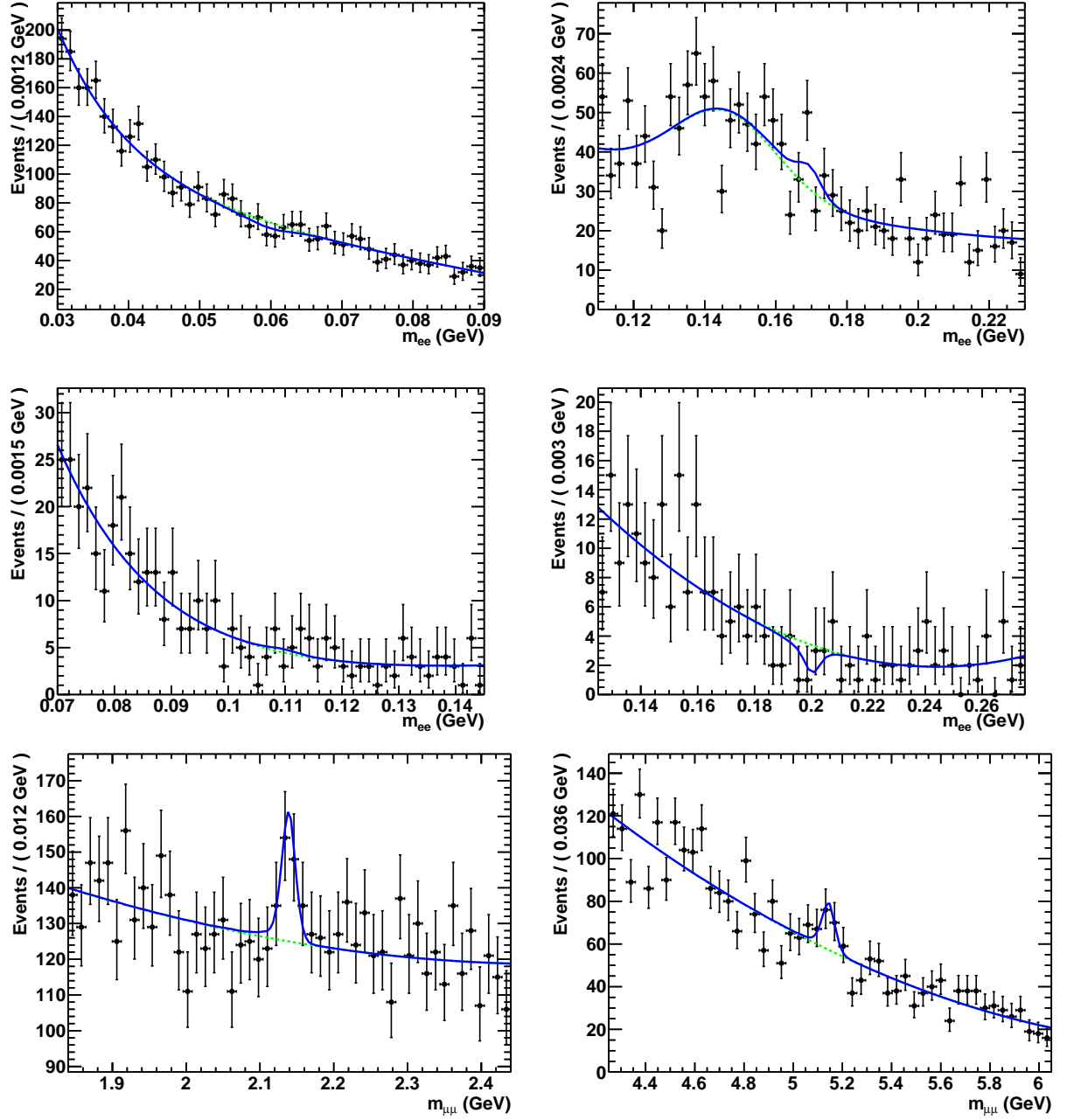


FIG. 3: Example of fits to (top) the dielectron mass with  $c\tau = 1$  mm; (middle left)  $c\tau = 10$  mm; (middle right)  $c\tau = 100$  mm; (bottom) the dimuon mass. The full fit is shown as a solid blue line, and the background as a dashed green line.